Quantificational logic and empty names

Andrew Bacon

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1 A Puzzle For Classical Quantificational Theory

Empty Names: Consider the sentence

1. There is something identical to Pegasus

On its most natural formalisation, $\exists xa = x$, this sentence translates to a theorem of classical logic (start with the axiom $a = a$ and then existentially generalise.)

Non Empty Names: Perhaps this is due to an incorrect formalisation or treatment of empty names. This thought does not get to the heart of the issue: the problem is not limited to empty names. Consider

2. Necessarily, there is something identical to Timothy Williamson.

This sentences most straightforward formalisation, $\Box\exists xa = x$, is a theorem provided we have a rule of necessitation (from a proof of ϕ from no premises infer $\Box\phi$) that ensures, minimally, that all the theorems of quantificational logic are necessary.

Necessitism: Perhaps this problem is due to an incorrect treatment of names altogether. The problem doesn't even have to trade on names, provided we allow open formulae to be theorems.

3. Necessarily, everything necessarily is identical to something.

Once again this is a theorem given the rule of necessitation and classical quantificational theory.

Choice: if we want to avoid these consequences either we reject classical quantification theory or the rule of necessitation.

Could we reject the rule of necessitation?

- Classical logic is not necessarily truth preserving.
- Classical logic does not even reliably lead to knowledge.
- The issue is verbal. (Necessitation deniers and non-standard logicians will agree about the truths and disagree only about what we call 'logic'. The logic of necessary truth preservation, for example, will be non-classical according to either disputant.)

2 Free Logic

How should we replace classical logic? In the first two cases we only used existential generalisation and the self identity axiom (some free logicians deny the latter, but we can run similar problems without using the self identity axiom.)

My favoured solution (which I believe is the earliest system, discovered by Kripke and Lambert) is to take a completely standard axiomatisation of classical logic with identity and replace universal instantiation (UI) with its universal closure (CUI):

UI $\forall x\phi \to \phi[t/x]$

CUI $\forall y(\forall x\phi \to \phi[y/x])$

This is called 'positive free logic'. Some people strengthen the logic to a 'negative free logic' by allowing more instances of UI. Important points to note

- CUI looks like it is actually at least as strong as UI. Not so: getting from CUI to UI requires an instance of UI.
- There are other more complicated axiomatisations of positive free logic floating around which has lead to the impression that free logic is unwieldy and that there is no agreement about what the correct logic is. Note then that (1) the described system is exactly as complicated as classical logic and (2) these variant systems have the same closed theorems (they tend to only differ about how open formulae are treated.[1])
- This formulation requires that the axioms for identity be present. Removing the identity axioms will remove some purely quantificational theorems; thus this axiomatisation is not conservative over its identity free fragment (unlike classical logic.)

Model Theory: the model theory for this logic is a very simple generalisation of classical model theory. One just relaxes the constraint that monadic predicates denote subsets of the domain (they may denote any old set), and similarly for $n$ary predicates, and one relaxes the constraint that names denote members of the domain and that variable assignments map variables into the domain. This is often called 'Meinongian model theory' because names receive denotations that are not in the domain of quantification.

3 Are Empty Names Intersubstitutable?

Within the free logic literature a lot of attention is devoted to the following question:

[1] I am talking only about the variant axiomatisations of positive free logic here. There is, of course, a substantial disagreement about whether the schema $\forall x\phi \to \phi[t/x]$ is valid for atomic ϕ.

Are atomic sentences involving empty names sometimes true, or are they always false/neither true nor false?

The view that atomic sentences containing empty names are never true motivates a slightly stronger logic than the one I have been endorsing.

My view is that atomic sentences containing empty names are sometimes true. However there is a lot of wriggle room concerning which sentences we class as atomic; presumably we can use the word 'atomic' in such a way that the thesis comes out true. To my mind the interesting question is not whether atomic sentences containing empty names are ever true, but whether empty names ever make an interesting semantic contribution to sentences in which they appear. The position that they don't make a semantic contribution might motivate the claim about atomic sentences, but it is a much more substantive claim.

The question I'd rather address is the following: can two distinct empty names have a different semantic profiles? On the assumption that they don't we should accept the following intersubstitutivity principle for empty names:

INTERSUBSTITUTIVITY If s and t are non-denoting terms then s and t are intersubstitutable for one another *salve veritate*.

INTERSUBSTITUTIVITY FORMAL $\forall x(x \neq s \wedge x \neq t) \rightarrow (\phi \rightarrow \phi[s/t])$.

Here ϕ need not be an extensional formula.

There are a variety of putative counterexamples to the intersubstitutivity principle, all of which suggest to me that empty names often do make a significant semantic contribution to sentences in which they appear.

1. INTENSIONAL TRANSITIVES: Botticelli painted Venus (but not Pegasus.)
2. PROPOSITIONAL ATTITUDES: Alice thought that Sherlock Holmes (but not Watson) was a detective.
3. COUNTERFACTUALS: Had Holmes and Watson existed Holmes (but not Watson) would have been a detective.
4. PREDICATE MODIFIERS: Pegasus is a mythological horse-god.

Each of these problems can be run without involving empty names.

4 The Semantic Problem

Recall the Meinongian model theory

- Model theory is a way of characterising *inferences*. Once we have a sound and complete model theory for a collection of inferences, we can use it to verify whether an argument is valid, whether a set of sentences is consistent or inconsistent and so on.

- The Meinongian model theory does not provide a *semantics* – it tells us nothing about what is true and false.

The semantic problem is the problem of specifying the truth conditions for a sentence that contains an empty name.

- No Meinongian model can provide the truth conditions for the sentence 'there is nothing identical to Pegasus.'
  - For this sentence to be true 'Pegasus' must be assigned an object that is not in the domain of the model.
  - But if the domain really includes everything there is (which it should, since it's supposed to be the intended model) there are no objects outside the domain.
- Even though no Meinongian model is the intended interpretation, we can still justify its usefulness for characterising the notion of a valid inference.
  - A variant of Kreisel's argument: If an inference is provable in PFL it is surely truth preserving over the class of all interpretations (Meinongian or not), from which it follows that it's truth preserving over the smaller class of Meinongian models. Finally by the completeness theorem it follows that the inference is provable in PFL. Thus we have a closed loop of entailments: a sequent is provable in PFL iff it is truth preserving in all interpretations, iff it is truth preserving in all Meinongian models.

Two observations

- Note that the most natural meaning to assign the name 'Pegasus' would be Pegasus itself.
  - 'Pegasus' refers to Pegasus
  - 'is a mythological horse-god' applies to Pegasus.
- This might seem incoherent at first because surely we are also committed to the following:
  - 'Pegasus' doesn't refer to anything. ('Pegasus' is an empty name.)
  - 'is a mythological horse-god' doesn't apply to anything. (Since there aren't any mythological horse-gods.)
- Note, however, that these pairs of claims are not inconsistent in positive free logic.
- For some verbs are existence entailing (e.g. 'jumps', 'walks', 'talks') other aren't ('imagines', 'draws', 'depicts'.) You can depict Pegasus without there being anything you're depicting.

My first thesis is that the verb 'refers' is not existence entailing (it belongs to the latter category.) Just as you can depict Pegasus without there being anything you're depicting, you can refer to Pegasus without there being anything you're referring to.

First Thesis: 'refers' is not existence entailing.

Assuming the first thesis, with a bit of care, it is possible to give something like a Tarskian compositional semantics for a first order language containing empty names, by appealing to claims of the form "Pegasus' refers to Pegasus', and similar claims.

5 The Metasemantic Problem

The metasemantic problem is this

> What is it about our use of the word 'Sherlock Holmes' that ensures that we refer to Sherlock Holmes, and not (say) Pegasus?

The causal account

- Something like the causal account of reference is surely true: when the use of a name can be causally traced back a number of initial baptismal speech acts, these speech acts play an important role in fixing the names referent. There might be no single baptismal speech act but a network of introductory uses.

- Whether or not this account is true to the letter should not matter for my purposes. A simple case is best for illustration:

  - When a name is introduced by a baptismal speech act of the form 'let 'a' denote the F', this act is appropriate when the conversational presuppositions entail that there is exactly one salient F.
  - When the background presuppositions are false (perhaps we sincerely believe that there is at least one horse-god, or we are engaging in a pretense) such a speech act still seems to be appropriate, and may even confer meaning to the name.
  - Once we learn that the presuppositions are false we can later report that, had those suppositions been true the thing we actually refer to with 'a' would be the thing that would have been F. (This

- In short the reason we pick out Holmes and not Pegasus with the name 'Sherlock Holmes' has to do with the counterfactual properties Holmes has which Pegasus doesn't. (Holmes, not Pegasus, would have been a detective living 221B Baker Street, had the presuppositions in which 'Sherlock Holmes' was introduced been true.)

A straightforward consequence of this view about reference fixing is there could have been things to which our empty names actually refer (even if these things don't actually exist.)

> Second Thesis: for every meaningful empty name, there could have been something to which it actually refers.

Worries:

- Given the 'Pegasus' refers to Pegasus and there could have been something 'Pegasus' actually refers to, it follows that Pegasus could have existed.
- But didn't Kripke refute this view? Kripke's argument is effectively that there are thousands of possible objects satisfying the properties associated with Pegasus in mythology, it seems absurd to think that our use of the word 'Pegasus' singles out one of these.
  - Firstly: according to the current view, you must do more than satisfy the properties associated by Greek mythology – you must be the horse-god that *would* have existed had the Greek myths been true.
  - Secondly: this argument over generates. According to the problem of the many there are an abundance of candidate referents even for names of existing macroscopic objects.
- What about names introduced within inconsistent suppositions?